Note on a reformulation of the strong cosmic censor conjecture based on computability

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Abstract

In this letter we provide a reformulation of the strong cosmic censor conjecture taking into account recent results on Malament–Hogarth space-times.

We claim that the strong version of the cosmic censor conjecture can be formulated by postulating that a physically reasonable space-time is either globally hyperbolic or possesses the Malament–Hogarth property. But it is known that a Malament–Hogarth space-time in principle is capable for performing non-Turing computations such as checking consistency of ZFC set theory.

In this way we get an intimate conjectured link between the cosmic censorship scenario and computability theory.

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1 Introduction

There is a remarkable recent interest in the physical foundations of computability theory and the Church–Turing Thesis. It turned out that algorithm and complexity theory, previously considered as very pure mathematical subjects, have a deep link with basic concepts of physics. At one hand now we can see that our deep and apparently pure mathematical notion of a Turing machine involves indirect preconceptions on space, time, motion and measurement. Hence it is straightforward to ask whether different choices of physical theories for modeling these things have some effect on our notions of computability or not. At the recent stage of affairs it seems there are striking changes on the whole structure of complexity and even computability theory if we move from classical physics to quantum or relativistic theories. Even variants of the Church–Turing Thesis cease to be valid in certain cases.

For instance, by taking quantum mechanics as our background theory, Calude and Pavlov have claimed in their recent paper that the famous Chaitin's Omega number, a typical non-computable real number, is enumerable via an advanced quantum computer [2] while Kieu has recently proposed an adiabatic quantum algorithm to attack Hilbert's tenth problem [8].

In the same fashion if we use general relativity theory, powerful gravitational computers can be built up which are also capable to break Turing's barrier. Malament and Hogarth proposed a class of space-times, now called as Malament–Hogarth space-times, admitting gravitational computers for non-Turing computations [6] [7]. Hogarth' construction uses anti-de Sitter space-time which is in the focus of recent investigations in high energy physics. In the same spirit the author and Németi have constructed another example by exploiting properties of the Kerr–Newman space-time [4]. This space-time is also relevant as the only possible final state of a collapsed, massive, slowly rotating star of small electric charge. A general introduction to the field is Chapter 4 of Earman's book [3].

On the other hand it is conjectured that these generalized computational methods are not significant from a computational viewpoint only but, in the case of quantum computers at least, they are also in connection with our most fundamental physical concepts such as the standard model and string theory [1] [10].

The natural question arises if the same is true for gravitational computers i.e., is there any pure physical characterization of the above mentioned Malament–Hogarth space-times? In this short paper we try to argue that these space-times also appear naturally in the strong cosmic censorship scenario. Namely we claim that space-times possessing powerful gravitational computers form the unstable borderline separating the allowed and not-allowed space-times by the strong cosmic censor conjecture (but these space-times are still considered as "physically relevant" i.e., are not ruled out by the strong cosmic censor).

If our considerations are correct then we can establish a hidden link betwix non-Turing computability and the most exciting open problem of classical general relativity.

2 Malament–Hogarth space-times

In this section we introduce the concept of a Malament–Hogarth space-time. As a motivation we mention that in these space-times, at least in theory, one can construct powerful gravitational computers capable for computations beyond the Turing barrier. A typical example for such a computation is checking of consistency of ZFC set theory.

Then we prove a basic property of Malament–Hogarth space-times namely they lack global hyperbolicity and distinguish the two main subclasses of them. Finally we provide three physically relevant examples possessing the Malament–Hogarth property.

Remember that the length of a non-spacelike, once continuously differentiable curve $\gamma : \mathbb{R} \to N$ in a pseudo-Riemannian manifold (N, h) is the integral

$$\|\gamma\| = \int\limits_{\gamma} \mathrm{d}\gamma = \int\limits_{\mathbb{R}} \sqrt{-h(\dot\gamma(t), \dot\gamma(t))}\, \mathrm{d}t = \int\limits_{\mathbb{R}} \sqrt{-|\dot\gamma(t)|^2_h}\, \mathrm{d}t$$

if exists. If this integral is unbounded we shall write $\|\gamma\| = \infty$. Furthermore we will be using the following standard terminology. Let (M, g) be a space-time which is a solution to Einstein's equation with a matter source represented by a stress-energy tensor T obeying the dominant energy condition on M. We will suppose this matter is *fundamental* in the sense that the associated

Einstein's equation (derived as a variation of T with respect to the metric g) can be put into the form of a quasilinear, diagonal, second order, hyperbolic system of partial differential equations. It is well-known that in this case (M, g) admits a well-posed initial value formulation (S, h, k) [5] [11]. Here S is a connected, spacelike hypersurface, $h = g|_S$ is the restricted Riemannian metric on S while k is the second fundamental form of (S, h) as embedded in (M, g).

Now consider the following class of space-times (cf. [4] but also [6][7]):

Definition 2.1 *Let (S, h, k) be an initial data set for Einstein's equation, with (S, h) a complete Riemannian manifold. Suppose a fundamental matter field is given represented by its stress-energy tensor T satisfying the dominant energy condition. Let (M, g) be a maximal analytical extension (if exists) of the unique maximal Cauchy development of the above initial data set.*

Then (M, g) is called a Malament–Hogarth space-time *if there is a future-directed timelike half-curve $\gamma_C : \mathbb{R}^+ \to M$ such that $\|\gamma_C\| = \infty$ and there is a point $p \in M$ satisfying* $\mathrm{im}\gamma_C \subset J^-(p)$. *The event $p \in M$ is called a* Malament–Hogarth event. $\diamond$

Note that if (M, g) is a Malament–Hogarth space-time, then there is a future-directed timelike curve $\gamma_O : [a, b] \to M$ from a point $q \in J^-(p)$ to p satisfying $\|\gamma_O\| < \infty$. The point $q \in M$ can be chosen to lie in the causal future of the past endpoint of γ_C.

Moreover the reason we require fundamental matter fields obeying the dominant energy condition, geodesically complete initial surfaces etc., is that we want to exclude the very artificial examples for Malament–Hogarth space-times.

The motivation is the following (for details we refer to [4]). Consider a Turing machine realized by a physical computer C moving along the curve γ_C of *infinite* proper time. Hence the physical computer (identified with γ_C) can perform arbitrarily long calculations in the ordinary sense. Being (M, g) a Malament–Hogarth space-time, there is an observer O following the curve γ_O (hence denoted by γ_O) of *finite* proper time such that he touches the Malament–Hogarth event $p \in M$ in finite proper time. But by definition $\mathrm{im}\gamma_C \subset J^-(p)$ therefore in p he can receive the answer for a *yes or no question* as the result of an *arbitrarily long* calculation carried out by the physical computer γ_C. This is because γ_C can send a light beam at arbitrarily late proper time to γ_O. Clearly the pair (γ_C, γ_O) is an *artificial computing system* i.e., a generalized computer in the sense of [4].

Imagine the following situation as an example. γ_C is asked to check all theorems of our usual set theory (ZFC) in order to check consistency of mathematics. This task can be carried out by γ_C since its world line has infinite proper time. If γ_C finds a contradiction, it can send a message (for example a light beam) to γ_O. Hence if γ_O receives a signal from γ_C *before* the Malament–Hogarth event p he can be sure that ZFC set theory is not consistent. On the other hand, if γ_O does not receive any signal before p then, *after* p, γ_O can conclude that ZFC set theory is consistent. Note that γ_O having finite proper time between the events $\gamma_O(a) = q$ (starting with the experiment) and $\gamma_O(b) = p$ (touching the Malament–Hogarth event), he can be sure about the consistency of ZFC set theory in finite (possibly very short) time. This shows that certain very general formulations of the Church–Turing thesis cannot be valid in the framework of classical general relativity [4].

One can raise the question if Malament–Hogarth space-times are relevant or not from a physical viewpoint. We put off this very important question for a few moments; instead we prove basic properties of Malament–Hogarth space-times by evoking Lemma 4.1 and Lemma 4.3 from [3]. These characteristics are also helpful in looking for realistic examples.

Proposition 2.2 *Let (M, g) be a Malament–Hogarth space-time with a timelike curve γ_C as above. Then (M, g) is not globally hyperbolic. Moreover, if $p \in M$ is a Malament–Hogarth event*

and $S \subset M$ is a connected spacelike hypersurface such that $\mathrm{im}\gamma_C \subset J^+(S)$ *then p is on or beyond $H^+(S)$, the future Cauchy horizon of S.*

Proof. Consider the point $q \in M$ such that $\gamma_C(0) = q$. If (M, g) was globally hyperbolic then (M, g) would be strongly causal and in particular $J^-(p) \cap J^+(q) \subset M$ compact. We know that $\mathrm{im}\,\gamma_C \subset J^-(p)$ hence in fact $\mathrm{im}\,\gamma_C \subset J^-(p) \cap J^+(q)$. Consequently its future (and of course, past) endpoint are contained in $J^-(p) \cap J^+(q)$ (cf. Lemma 8.2.1 in [11]). However γ_C is a causal curve with $\|\gamma_C\| = \infty$ hence it is future inextendible i.e., has no future endpoint. But this is impossible hence $J^-(p) \cap J^+(q)$ cannot be compact or strong causality must be violated within this set leading us to a contradiction.

Secondly, assume $p \in D^+(S) \setminus \partial D^+(S)$ i.e., p is an interior point of the future domain of dependence of S. Then there is an $r \in D^+(S)$ chronologically preceded by p (with respect to some time function assigned to the Cauchy foliation of $D^+(S)$). Letting $N := J^-(r) \cap J^+(S)$ then $N \subset D^+(S)$ hence $(N, g|_N)$ is a globally hyperbolic space-time containing the Malament–Hogarth event p and the curve γ_C. Consequently we can proceed as above to arrive at a contradiction again. $\diamond$

The proof of this proposition provides us a characterization of Malament–Hogarth space-times.

Proposition 2.3 *Let (M, g) be a Malament–Hogarth space-time with $p \in H^+(S) \subset M$ a Malament–Hogarth event. Consider a timelike curve γ_C as above with* $\mathrm{im}\gamma_C \subset J^+(S)$. *Then either $\overline{J^-(p) \cap S}$ is non-compact or strong causality is violated at $p \in M$.*

Proof. We just have to repeat the pattern of the proof of the previous proposition.

Assume $\overline{J^-(p) \cap S}$ is compact. Then this set is bounded with respect to h in the geodesically complete (S, h). As we have seen, there always exists a timelike curve $\gamma_O : [a, b] \to M$ of finite proper time with $\gamma_O(a) \in S$ and $\gamma_O(b) = p$ ("observer"). Hence by boundedness all points of $\overline{J^-(p) \cap S}$ can be joined with p by a causal curve of finite length therefore our assumption implies that $J^-(p) \cap J^+(S)$ is compact, too. But the complete timelike curve $\gamma_C : \mathbb{R}^+ \to M$ with $\gamma_C(0) \in S$ is future inextendible and satisfies $\mathrm{im}\gamma_C \subset J^-(p) \cap J^+(S)$ hence there must exist a point $p' \in J^-(p) \cap J^+(S)$ at which strong causality is violated (again by Lemma 8.2.1 of [11]). But this point cannot exists in the interior of $M = D^+(S)$ because this part is globally hyperbolic therefore must have $p' \in H^+(S)$ and it is a Malament–Hogarth event in this case as easily seen. Consequently $p = p'$ holds.

In the opposite way if strong causality is valid in (M, g) then the same is true for the portion $\overline{J^-(p) \cap J^+(S) \subset M}$ with $\mathrm{im}\gamma_C \subset J^-(p) \cap J^+(S)$. But in this case this set cannot be compact implying $\overline{J^-(p) \cap S}$ is non-compact taking into account geodesic completeness of (S, h). $\diamond$

Now we can turn our attention to the existence of physically relevant examples of space-times containing gravitational computers. Proposition 2.3 indicates that the class of Malament–Hogarth space-times can be divided into two major subclasses: the first one contains space-times in which an infinite, non-compact portion of a spacelike submanifold is visible from some event. The question is if certain members of these space-times obey some energy condition or not with some standard matter content (in this case a space-time is considered as "physical"). The answer is yes: examples for such space-times are provided by the following proposition.

Proposition 2.4 *Kerr space-time and the universal covering space of anti-de Sitter space-time are Malament–Hogarth space-times.*

Proof. In the case of the universal cover of anti-de Sitter space-time (which is a vacuum space with non-vanishing cosmological constant) this has been proved in [6] while the case of Kerr space-time (this is a vacuum space-time with vanishing cosmological constant) has been worked out in [4]. $\diamond$

(We remark that Reissner–Nordström space-time, very similar to the Kerr one, also admits Malament–Hogarth events). The second subclass consists of those space-times which violate strong causality in a suitable way. A typical example is presented in the next proposition.

Proposition 2.5 *Consider a particular maximal analytical extension of Taub–NUT space-time. Then this space-time possesses the Malament–Hogarth property and Malament–Hogarth events are situated along the Cauchy horizons of this space-time.*

Proof. Take the space $S^3 \times \mathbb{R}$ equipped with the Taub–NUT metric g on it (cf. [5], pp. 170-178):

$$\mathrm{d}s^2 = -\frac{\mathrm{d}t^2}{U(t)} + 4a^2U(t)(\mathrm{d}\psi + \cos\Theta\mathrm{d}\phi)^2 + (t^2+a^2)(\mathrm{d}\Theta^2 + \sin^2\Theta\mathrm{d}\phi^2).$$

This is a spherically symmetric vacuum metric and

$$U(t) = -1 + \frac{2(mt+a^2)}{a^2+t^2}, \qquad m, a \text{ are positive constants.}$$

Moreover (ϕ, Θ, ψ) are the Euler angles on S^3 i.e., $0 \leq \psi \leq 4\pi$, $0 \leq \Theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. The metric is singular at the zeros of U equal to $t_\pm = m \pm \sqrt{m^2+a^2}$ hence apparently we have to make the restriction $t_- < t < t_+$. The time orientation is fixed such that t increases. However introducing the new variable

$$\psi' := \psi + \frac{1}{2a}\int\limits_{t_0}^{t} \frac{\mathrm{d}\tau}{U(\tau)} \pmod{4\pi},$$

we can rewrite the metric (also denoted by g) as

$$\mathrm{d}s^2 = -4a\mathrm{d}t(\mathrm{d}\psi' + \cos\Theta\mathrm{d}\phi) + 4a^2U(t)(\mathrm{d}\psi' + \cos\Theta\mathrm{d}\phi)^2 + (t^2+a^2)(\mathrm{d}\Theta^2 + \sin^2\Theta\mathrm{d}\phi^2) \qquad (1)$$

and now we can allow $-\infty < t < \infty$ without difficulty but the resulting analytical extension lacks global hyperbolicity. Indeed, the surfaces given by $t = t_\pm$, diffeomorphic to S^3, represent Cauchy horizons (and coincide if $a = 0$). There is another analytical extension of Taub–NUT space, too. For details see [5].

Now we can proceed as follows. Consider a smooth curve $\alpha : \mathbb{R}^+ \to S^3 \times \mathbb{R}$ given by

$$\beta(s) := \left(\Theta_0, \phi_0,\ \psi_0 + \frac{1}{2a}\int\limits_0^s \frac{e^{-u}}{U(t(u))}\mathrm{d}u,\ t(s)\right), \quad \text{where } t(s) = t_+ - e^{-s} \qquad (2)$$

with respect to the coordinate system (1). The constants Θ_0, ϕ_0, ψ_0 are fixed. This curve is clearly future directed and runs in the region $t_+ - 1 \leq t < t_+$ which is inside the original globally hyperbolic region $t_- < t < t_+$ if $2\sqrt{m^2+a^2} > 1$. Its tangent vector field is

$$\frac{\mathrm{d}\alpha}{\mathrm{d}s} = \dot{\alpha}(s) = \frac{e^{-s}}{2aU(t(s))}\frac{\partial}{\partial\psi'} + e^{-s}\frac{\partial}{\partial t}$$

of pointwise length $|\dot{\alpha}(s)|^2_g = -e^{-2s}/U(t(s))$ by using (1). But for $0 \leq s < \infty$ that is, for $t_+ - 1 \leq t(s) < t_+$ this is clearly negative showing that our curve is not only future directed but $|\dot{\alpha}(s)|^2_g < 0$ that is, timelike as well. Its length is

$$\|\alpha\| = \int\limits_0^\infty \frac{e^{-s}}{\sqrt{U(t(s))}}\,\mathrm{d}s \geq \int\limits_0^\infty e^{-s}\,\mathrm{d}s = \infty$$

hence is complete. Clearly, α spirals around the Cauchy surface $t = t_+$ infinitely many times while approaching it.

Furthermore this α can be deformed into a future directed, timelike complete geodesic, too. To see this, consider the sequence of points $\{\alpha(s_n) \mid n \in \mathbb{N}\}$ for a suitable monotonly increasing sequence of points $s_n \in \mathbb{R}^+$. Since the portion of the extended Taub–NUT space-time containing α is globally hyperbolic and $\alpha(s_n) << \alpha(s_k)$ if $n < k$ we can find future directed timelike geodesic segments γ_n connecting $\alpha(s_n)$ with $\alpha(s_{n+1})$. If the partition is dense enough, γ_n's are close to α_n's where $\alpha_n := \{\alpha(s) \mid s \in [s_n, s_{n+1}]\}$. Because geodesics have extremal length among curves, we can write for an at least piecewise smooth geodesic $\gamma_C : \mathbb{R}^+ \to S^3 \times \mathbb{R}$ that

$$\|\gamma_C\| = \sum_{n\in\mathbb{N}} \|\gamma_n\| \geq \sum_{n\in\mathbb{N}} \|\alpha_n\| = \|\alpha\| = \infty.$$

In other words our future directed timelike geodesic γ_C just constructed is complete and spirals around the Hopf-circle $S^1 := \{(\phi_0, \Theta_0, \psi', t_+) \mid 0 \leq \psi' \leq 4\pi\}$ (here ϕ_0 and Θ_0 coincide with constants chosen in (2)). This circle is on the Cauchy horizon $t = t_+$ of the maximally extended Taub–NUT space-time. Now it is straightforward that at all the events $p \in S^1$ strong causality is violated by γ_C (or α).

Finally consider an even more simple smooth curve $\beta : [0, 1] \to S^3 \times \mathbb{R}$ which is given with respect to (1) as

$$\beta(s) := (\Theta_0, \phi_0, \psi_0 + s, t(s)), \quad \text{where } t(s) = t_+ - 1 + s.$$

Hence

$$\dot{\beta}(s) = \frac{\partial}{\partial\psi'} + \frac{\partial}{\partial t}$$

yielding $|\dot{\beta}(s)|^2_g = 4a(aU(t(s)) - 1)$ via (1). Consequently for $0 \leq s \leq 1$ i.e., for $t_+ - 1 \leq t(s) \leq t_+$ one again has $|\dot{\beta}(s)|^2_g < 0$ if $a < 1$ demonstrating that this curve is also future directed and timelike of finite length:

$$\|\beta\| = 2\sqrt{a}\int\limits_0^1 \sqrt{1 - aU(t(s))}\,\mathrm{d}s \leq 2\sqrt{a} < \infty.$$

Proceeding as above we can deform this curve into an at least piecewise smooth timelike geodesic γ_O also of finite length. One can see that $\gamma_C(0) = \gamma_O(0)$ furthermore γ_O intersects γ_C infinitely many times hence the point $p = \gamma_O(1) = (\Theta_0, \phi_0, \psi_0 + 1, t_+) \in S^1 \subset S^3$ is a Malament–Hogarth event. Consequently we can interpret γ_C as a "computer" required in the definition of Malament–Hogarth space-times moreover γ_O as an "observer". This shows the Malament–Hogarth property of Taub–NUT space (together with the technical condition $0 < a < 1 < 2\sqrt{m^2 + a^2}$ but this is certainly avoidable by using better curves). $\diamond$

After getting the feel of Malament–Hogarth space-times we move to the next section where their relation with the strong cosmic censorship is pointed out.

3 The strong cosmic censor conjecture

Cosmic censorship is being used to rule out space-times where causality is violated because of the presence of "naked singularities". Since nowadays we are unable to grasp the notion of a naked singularity in its full generality we are forced to use some other indirect characteristics to remove those space-times where causality breaks down from the class of physically relevant ones. The most straightforward notion is global hyperbolicity because in this case all events can be predicted from an initial data set fixed in advance along a Cauchy surface. However this restriction is too strong: physically relevant examples like anti-de Sitter, Taub–NUT and even the Kerr–Newman space-times would considered as "wrong" by requiring simply global hyperbolicity. Consequently the recent versions of the strong cosmic censor conjecture are formulated by postulating global hyperbolicity for "physically relevant" space-times but also providing a "list" of sporadic examples lacking global hyperbolicity nevertheless considered as "physically relevant".

A more or less up-to-date formulation is given in [11] on p. 305.

Conjecture 3.1 (standard formulation of the strong cosmic censor conjecture) *Let (S, h, k) be an initial data set for Einstein's equation, with (S, h) a complete Riemannian manifold. Suppose a fundamental matter field is given represented by its stress-energy tensor T satisfying the dominant energy condition. Then, if the maximal Cauchy development of this initial data set is extendible, for each $p \in H^+(S)$ in any extension, either strong causality is violated at p or $\overline{J^-(p) \cap S}$ is non-compact.*

It is quite surprising that in light of Proposition 2.3 Malament–Hogarth events have exactly the same properties as points have on the Cauchy horizons in the above formulation. In other words the content of Proposition 2.3 is that Malament–Hogarth property implies the behaviour for the Cauchy horizon required by the strong cosmic censor conjecture.

To see precise equivalence we should establish a converse to Proposition 2.3. That is, if a space-time is non-globally hyperbolic and for its events $p \in H^+(S)$ either $\overline{J^-(p) \cap S}$ is non-compact or strong causality is violated at p then is (M, g) a Malament–Hogarth space-time? The answer is certainly no because the conditions are insufficient. For instance we should know something on the length of the Cauchy development in question.

However the statement "for each $p \in H^+(S)$ in any extension, either strong causality is violated at p or $\overline{J^-(p) \cap S}$ is non-compact" is being used only to incorporate the classical examples like Kerr–Newman or Taub–NUT into the allowed class of space-times. But we have seen that these examples possess the Malament–Hogarth property. In other words the Malament–Hogarth property appears as a unifying way to enumerate the important and known sporadic examples in the strong cosmic censor conjecture considered as "still physically relevant" although they are non-globally hyperbolic i.e., which posses an initial value formulation but their maximal Cauchy developments are extendible. Hence we cannot resist the temptation to reformulate the above conjecture as follows.

Conjecture 3.2 (reformulation of the strong cosmic censor conjecture) *Let (S, h, k) be an initial data set for Einstein's equation, with (S, h) a complete Riemannian manifold. Suppose a fundamental matter field is given represented by its stress-energy tensor T satisfying the dominant energy condition. Then, if the maximal Cauchy development of this initial data set is extendible, this extension is a Malament–Hogarth space-time.*

We can examine the strong cosmic censor scenario from a naive stability point of view as well. It is well-known (cf. Theorem 8.3.14 of [11]) that a globally hyperbolic space-time is stable causal. But stable causality is a stable property under small perturbations of the metric (here "small" is understood as follows: if g is the original metric and $g' = g + h$ is its perturbation then for the tensor field h we have $\|h\|$ small in an appropriate Sobolev norm on the space of metrics over a fixed manifold M). Consequently global hyperbolicity seems to be a stable property hence globally hyperbolic space-times apparently form the "interior" of the set of "physically reasonable" space-times allowed by the strong cosmic censor.

On the other hand, it is known or at least conjectured that space-times like (maximal extensions of) Kerr or Taub–NUT are unstable in the sense that small perturbations of these metrics deform the Cauchy horizons of these space-times into a real curvature singularity. In the case of black holes this is called mass inflation (e.g. cf. [9]). Therefore these space-times may be considered as unstable in an appropriate fashion. Or, turning the coin, we can intuitively say that the Malament–Hogarth property is an unstable property of "physically relevant" space-times and form the "borderline" between allowed and non-allowed space-times by the strong cosmic censorship. It is quite interesting that exactly this unstable thin class admits gravitational computers providing non-Turing computations.

The whole conjectured situation is sketched in the following figure.

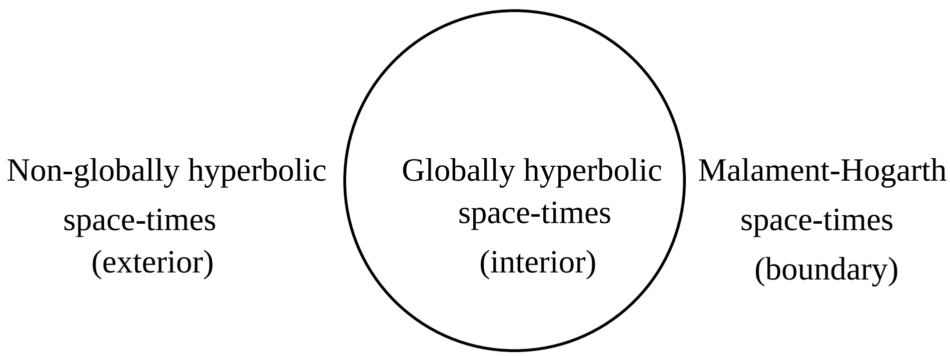

Figure 1. The strong cosmic censorship scenario

4 Concluding remarks

As a conclusion we have to emphasize again that our considerations require future work for example it is important to understand if other marginal space-times (from the point of view of the strong cosmic censor) admit the Malament–Hogarth property or not.

Another important problem is to understand the stability properties of Malament–Hogarth space-times mentioned above and see if these indeed form a kind of boundary in a strict functional analytic sense for physically relevant metrics over a fixed manifold.

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